Neutrino masses and mixing in A_4 models with three Higgs doublets

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We study neutrino masses and mixing in the context of flavor models with A_4 symmetry, three scalar doublets in the triplet representation, and three lepton families. We show that there is no representation assignment that yields a dimension-five mass operator consistent with experiment. We then consider a type-I seesaw with three heavy right-handed neutrinos, explaining in detail why it fails, and showing with a numerical example that agreement with the present neutrino oscillation data can be recovered with the inclusion of dimension-three heavy neutrino mass terms that break softly the A_4 symmetry.

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I. INTRODUCTION

One can attempt to explain the structure of neutrino masses and mixing by building full theories subject to discrete symmetries (for recent reviews, see e.g. Refs. [1–4]). As an alternative, one may consider effective operators in the low-energy limit and their remnant symmetries, without recourse to the full theory [5]. In either case, it is important to ensure that the vacuum structure chosen for the scalar sector of the theory does indeed correspond to a global (and not merely a local) minimum. Recently [6], the global minima of theories with three Higgs doublets, Φ_k $(k = 1, 2, 3)$, in triplet representations of A_4 [7] or S_4 have been identified. It turns out that the allowed alignments for the vacuum expectation values (VEV) in A_4 are [6]

$$\begin{aligned} &v\,(1,0,0),\\ &v\,(1,1,1),\\ &v\,(\pm1,\eta,\eta^*)\ \text{with}\ \ \eta=e^{i\pi/3},\\ &v\,(1,e^{i\alpha},0)\ \text{with any phase}\ \alpha. \end{aligned} \tag{1}$$

Permutations of these VEV are still global minima; other solutions of the stationarity conditions are not. The quark sector of such theories was explored by us in Ref. [8]. It was shown that, at tree level, there is no consistent theory with only three families of standard quark fields that can explain the fact that the quark masses and the Cabibbo-Kobayashi-Maskawa CP-violating phase are nonvanishing.

In this article, we extend the analysis of Ref. [8] to the neutrino sector, assuming that neutrino masses are generated through a low-energy mass operator in an effective theory. We consider models with three Higgs doublets Φ_k in a triplet representation of A_4, so that the only possible VEV structures are those in Eqs. (1).

In Sec. II we recall the features that apply to the charged-lepton Yukawa matrices in A_4 [8]. Then, we turn to the neutrino sector. In Sec. III A, we address the question of whether it is possible to get a consistent picture in an effective theory with the following particle content: three scalar doublets in a triplet representation of A_4; three left-handed lepton doublets L_L and three right-handed charged leptons ℓ_R in any representation of A_4. We conclude that, although an A_4-invariant dimension-five effective operator $(L_L\Phi)(L_L\Phi)$ can always be built, it is not possible to assign suitable A_4 representations to the fields in order to obtain viable (nondegenerate and nonvanishing) mass spectra, with the VEV alignments given in Eq. (1) and neutrino masses arising solely from the effective operator. As a result, in such a minimal A_4 framework, none of the standard seesaw mechanisms, usually invoked to give small masses to neutrinos, is consistent with the experimental data. With the aim of identifying what features need to be corrected when building a complete viable model, in Sec. III B we discuss in detail type I (type III) seesaw models [1] that contain a minimal particle content, namely, three generations of left-handed lepton doublets and right-handed charged-lepton singlets, and three right-handed neutrino singlets ν_R (fermion triplets Σ_R). Since, at the Lagrangian level, the A_4 flavour group structure is the same for both seesaw cases, so are the conclusions. Clearly, the above minimal setup is not sufficient to build a consistent A_4 flavor model that leads to nonzero nondegenerate charged-lepton and neutrino masses, and to the correct leptonic mixing. A viable numerical example based on the soft breaking of the A_4 symmetry is then presented in Sec. IV. Our conclusions are briefly summarized in Sec. V.

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[1] We do not consider here the type II seesaw scenario since it involves triplet scalars, thus making, in general, the identification of the global minima of the scalar potential extremely hard.

II. A_4 AND CHARGED LEPTONS

The A_4 group has four irreducible representations: three singlets $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, and one triplet $\mathbf{3}$. Here we adopt the basis used in Ref. [8] for the generators of the group, and place the three Higgs doublets in the triplet representation, $\Phi \sim \mathbf{3}$. The conclusions reached in Table II of Ref. [8] for the down-type quarks also hold for charged leptons. In particular, requiring nonvanishing nondegenerate charged-lepton masses forces the VEV alignments to be $v(1,1,1)$ or $v(\pm 1, \eta, \eta^*)$, restricting the possible representation assignments to the cases listed in Table I.

L_L	ℓ_R
$\mathbf{3}$	$\mathbf{3}$
$\mathbf{3}$	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$
$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$

TABLE I. Possible representations of the left-handed lepton doublet (L_L), and right-handed charged-lepton singlets (ℓ_R), which lead to nonvanishing nondegenerate charged lepton masses, when the three Higgs doublets are in a triplet representation $\mathbf{3}$.

III. A_4 AND MAJORANA NEUTRINOS

If neutrinos were Dirac particles, with masses arising from Yukawa couplings with Φ_k, our conclusions would be the same as those reached in Ref. [8] for the up-type quarks. Clearly, in that case, the existence of one massless neutrino or the lack of leptonic CP violation would not contradict current experiments [9]. In this work, however, we focus on Majorana neutrinos whose masses are given by some seesaw mechanism or else are generated effectively by integrating out unspecified heavy degrees of freedom.

A. Effective dimension-5 operator for neutrino masses

There is only one dimension 5 operator made out of the SM fields which respects the SM gauge symmetry [10],

$$\mathcal{L}_{\text{eff}} = \left(\bar{L}_L \tilde{\Phi}\right) K \left(\tilde{\Phi}^T L_L^c\right) + \text{H.c.}, \tag{2}$$

where K is a complex symmetric matrix, and $\tilde{\Phi} = i\sigma_2 \Phi$. Since we are only interested in the group structure, we will only refer to $L_L L_L$ and $\tilde{\Phi}\tilde{\Phi}$.

If $L_L \sim \mathbf{3}$ we can form the symmetric bilinears

$$(L_L \otimes L_L)_{\mathbf{1}} = L_1^2 + L_2^2 + L_3^2, \tag{3}$$
$$(L_L \otimes L_L)_{\mathbf{1}'} = L_1^2 + \omega^2 L_2^2 + \omega L_3^2, \tag{4}$$
$$(L_L \otimes L_L)_{\mathbf{1}''} = L_1^2 + \omega L_2^2 + \omega^2 L_3^2, \tag{5}$$
$$(L_L \otimes L_L)_{\mathbf{3}s} = 2(L_2 L_3, L_3 L_1, L_1 L_2), \tag{6}$$

where $\omega = e^{2i\pi/3}$. Similar combinations hold for $\tilde{\Phi}\tilde{\Phi}$. Forming all combinations that can lead to a singlet, we find

$$\begin{aligned}\mathcal{L}_{\text{eff}} = {} & \lambda_1 (L_1^2 + L_2^2 + L_3^2)(\tilde{\Phi}_1^2 + \tilde{\Phi}_2^2 + \tilde{\Phi}_3^2) \\ & + \lambda_2 (L_1^2 + \omega^2 L_2^2 + \omega L_3^2)(\tilde{\Phi}_1^2 + \omega \tilde{\Phi}_2^2 + \omega^2 \tilde{\Phi}_3^2) \\ & + \lambda_3 (L_1^2 + \omega L_2^2 + \omega^2 L_3^2)(\tilde{\Phi}_1^2 + \omega^2 \tilde{\Phi}_2^2 + \omega \tilde{\Phi}_3^2) \\ & + \lambda_4 (L_2 L_3 \tilde{\Phi}_2 \tilde{\Phi}_3 + L_3 L_1 \tilde{\Phi}_3 \tilde{\Phi}_1 + L_1 L_2 \tilde{\Phi}_1 \tilde{\Phi}_2), \end{aligned} \tag{7}$$

where the $\lambda_1 \cdots \lambda_4$ are complex parameters.

After spontaneous symmetry breaking the fields get VEV $\langle \Phi_k \rangle = v_k$, and the elements of the symmetric matrix K become

$$\begin{aligned} k_{11} &= \lambda_1 (v_1^{*2} + v_2^{*2} + v_3^{*2}) + \lambda_2 (v_1^{*2} + \omega^2 v_2^{*2} + \omega v_3^{*2}) \\ & \quad \lambda_3 (v_1^{*2} + \omega v_2^{*2} + \omega^2 v_3^{*2}), \\ k_{22} &= \lambda_1 (v_1^{*2} + v_2^{*2} + v_3^{*2}) + \lambda_2 \omega (v_1^{*2} + \omega^2 v_2^{*2} + \omega v_3^{*2}) \\ & \quad \lambda_3 \omega^2 (v_1^{*2} + \omega v_2^{*2} + \omega^2 v_3^{*2}), \\ k_{33} &= \lambda_1 (v_1^{*2} + v_2^{*2} + v_3^{*2}) + \lambda_2 \omega^2 (v_1^{*2} + \omega^2 v_2^{*2} + \omega v_3^{*2}) \\ & \quad \lambda_3 \omega (v_1^{*2} + \omega v_2^{*2} + \omega^2 v_3^{*2}) \\ k_{12} &= \frac{1}{2} v_1^* v_2^* \lambda_4, \quad k_{13} = \frac{1}{2} v_1^* v_3^* \lambda_4, \quad k_{23} = \frac{1}{2} v_2^* v_3^* \lambda_4. \end{aligned} \tag{8}$$

It was shown in Ref. [8] that only the VEV alignments $(1,1,1)$ and $(\pm 1, \eta, \eta^*)$ [2] lead to non-vanishing charged lepton masses. With $(1,1,1)$, we find

$$K = \frac{1}{2} \begin{pmatrix} 6\lambda_1 & \lambda_4 & \lambda_4 \\ \lambda_4 & 6\lambda_1 & \lambda_4 \\ \lambda_4 & \lambda_4 & 6\lambda_1 \end{pmatrix}, \tag{9}$$

meaning that $KK^\dagger$ has a doubly degenerate eigenvalue $9|\lambda_1|^2 + |\lambda_4/2|^2 - 3\text{Re}(\lambda_1 \lambda_4^*)$ and a third eigenvalue $9|\lambda_1|^2 + |\lambda_4|^2 + 6\text{Re}(\lambda_1 \lambda_4^*)$. With $(\pm 1, \eta, \eta^*)$ the matrix K is slightly different, but the eigenvalues of $KK^\dagger$ are the same with $\lambda_1 \to \lambda_3$. So, the effective dimension-5 term with A_4 symmetry, $\Phi \sim \mathbf{3}$, and $L_L \sim \mathbf{3}$ is ruled out.

Finally, we turn to the possibility that L_L is a singlet of A_4. We know from Table I that the only choice compatible with nonzero non-degenerate charged lepton masses is $L_L \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\ell_R \sim \mathbf{3}$. The $L_L L_L$ group structures obtainable when $L_L \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ are contained in the right-hand side of Eq. (A.5). Entries with $\mathbf{1}$ couple to the $\tilde{\Phi}\tilde{\Phi}$ combination $(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}}$. This can be read from Eq. (3) by changing $L_L \to \tilde{\Phi}$, which after spontaneous symmetry breaking leads to $(v_1^{*2} + v_2^{*2} + v_3^{*2})$. Similarly, entries with $\mathbf{1}'$ in Eq. (A.5) couple to the $(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}''}$ combination $(v_1^{*2} + \omega v_2^{*2} + \omega^2 v_3^{*2})$. Finally, entries with $\mathbf{1}''$ in Eq. (A.5) couple to the $(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}'}$ combination $(v_1^{*2} + \omega^2 v_2^{*2} + \omega v_3^{*2})$.

For the VEV $(1,1,1)$, the above combinations give $v_1^{*2} + v_2^{*2} + v_3^{*2} = 3$, $v_1^{*2} + \omega v_2^{*2} + \omega^2 v_3^{*2} = 0$, and

[2] Henceforth, we assume without loss of generality that $v = 1$.

$v_1^{*2}+\omega^2 v_2^{*2}+\omega v_3^{*2}=0$. In this case, the matrix K is of the form

$$K = 3\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 \\ 0 & \lambda_2 & 0 \end{pmatrix}. \qquad (10)$$

The matrix $KK^\dagger$ has a doubly degenerate eigenvalue $3|\lambda_2|^2$, and a third eigenvalue $3|\lambda_1|^2$. Similarly, for the VEV $(\pm 1, \eta, \eta^*)$, the relevant VEV combinations are 0, 3, and 0, respectively, with

$$K = 3\begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix}. \qquad (11)$$

Again, $KK^\dagger$ has a doubly degenerate eigenvalue $3|\lambda_2|^2$, and a third eigenvalue $3|\lambda_1|^2$. As a result, all cases are ruled out.

B. Type I and type III seesaw

We consider first the type I [11] seesaw mechanism with $n_R = 3$ right-handed neutrino fields. The relevant Lagrangian is

$$-\mathcal{L}_{\mathrm{I}} = \bar{L}_L \sum_{k=1}^{3} Y_{\ell,k}\Phi_k \ell_R + \bar{L}_L \sum_{k=1}^{3} Y^*_{\nu,k}\tilde{\Phi}_k \nu_R + \frac{1}{2}\bar{\nu}_R M_R \nu_R^c + \text{H.c.}, \qquad (12)$$

where $L_L = (\ell_L, \nu_L)^T$, ℓ_R, and ν_R are vectors in the three dimensional generation spaces of left-handed doublets, right-handed charged-lepton singlets, and right-handed neutrino singlets, respectively. For each scalar doublet Φ_k there is a charged-lepton Yukawa matrix $Y_{\ell,k}$, and a neutrino Yukawa matrix $Y_{\nu,k}$. After the spontaneous symmetry breaking we obtain the mass terms

$$-\mathcal{L}_{\mathrm{I}} = \bar{\ell}_L m_\ell \ell_R + \bar{\nu}_L m_D \nu_R + \frac{1}{2}\bar{\nu}_R M_R \nu_R^c + \text{H.c.}, \qquad (13)$$

where

$$m_\ell = \sum_{k=1}^{3} Y_{\ell,k} v_k, \qquad m_D = \sum_{k=1}^{3} Y^*_{\nu,k} v_k^*, \qquad (14)$$

and M_R is a symmetric matrix. To correctly reproduce the light neutrino masses, the eigenvalues of M_R should be much larger than $(|v_1|^2+|v_2|^2+|v_3|^2)^{1/2}$. Integrating out the heavy right-handed Majorana fields, the low-energy effective Lagrangian becomes

$$-\mathcal{L}_{\mathrm{eff}} = \bar{\ell}_L m_\ell \ell_R + \frac{1}{2}\nu_L^T C m_\nu \nu_L + \text{H.c.}, \qquad (15)$$

and the light neutrinos acquire an effective mass

$$m_\nu = -m_D M_R^{-1} m_D^T. \qquad (16)$$

In the basis where the charged-lepton mass matrix is diagonal,

$$m_\ell = \mathrm{diag}\,(m_e, m_\mu, m_\tau), \qquad (17)$$

the neutrino mass matrix m_ν is diagonalized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [12] leptonic mixing matrix U as

$$U^T m_\nu U = \mathrm{diag}\,(m_1, m_2, m_3), \qquad (18)$$

where m_i are the light neutrino masses.

We will try to assign the lepton fields to A_4 representations, subject to the following constraints:

1. The matrix M_R corresponding to the heavy Majorana fields cannot have a zero eigenvalue;

2. The tree-level masses for the charged leptons cannot vanish or be degenerate.

The first condition forces the right-handed neutrino fields to be in one of the following three representations. One can have $\nu_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$, and

$$M_R = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \qquad (19)$$

where $\times$ represents an independent complex entry. Alternatively, one can have $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, and

$$M_R = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}. \qquad (20)$$

Finally, if $\nu_R \sim \mathbf{3}$, then

$$M_R = M\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (21)$$

where M is an arbitrary complex number. This corresponds to degenerate heavy neutrinos. Other combinations are ruled out by our first requirement. For example, choosing $\nu_R \sim (\mathbf{1}', \mathbf{1}', \mathbf{1}'')$, one finds

$$M_R = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \qquad (22)$$

leading to one massless "heavy" neutrino.

Before proceeding, let us look back at Eq. (16). Because $\det M_R \neq 0$, the existence (or absence) of massless light neutrinos depends on the nature of m_D. If

$\det m_D = 0$ ($\det m_D \neq 0$), then $\det m_\nu = 0$ ($\det m_\nu \neq 0$). As a result, we will consider the constraints coming from charged leptons in each of these two cases, separately. Notice that, regardless of the ν_R representation, no case with both L_L and ℓ_R in singlet representations is possible because $\Phi \sim \mathbf{3}$, leading to $m_\ell = 0$. Similarly, the cases where both L_L and ν_R are in singlet representations are excluded because they lead to $m_D = 0$ and, through Eq. (16), to $m_\nu = 0$. The remaining cases are listed in Table II.

Case	L_L	ν_R	ℓ_R	Neutrino masses
i)	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	2 degenerate
ii)	$\mathbf{3}$	$\mathbf{3}$	$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$	2 degenerate
iii)	$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$	$\mathbf{3}$	$\mathbf{3}$	2 degenerate
iv)	$\mathbf{3}$	$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$	$\mathbf{3}$	2 degenerate
v)	$\mathbf{3}$	$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$	$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$	2 degenerate
vi)	$\mathbf{3}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$\mathbf{3}$	2 massless
vii)	$\mathbf{3}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1}',\mathbf{1}'')$	2 massless

TABLE II. Possible representations of the left-handed lepton doublet (L_L), the right-handed neutrino singlets (ν_R), and right-handed charged-lepton singlets (ℓ_R), when the three Higgs doublets are in a triplet representation $\mathbf{3}$.

1. Nonvanishing neutrino masses

We start by looking at the case in which $\nu_R \sim \mathbf{3}$. From Table II one concludes that there are three possibilities which may lead to nonvanishing neutrino and charged-lepton masses: i) $L_L \sim \mathbf{3}$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$; ii) $L_L \sim \mathbf{3}$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim (\mathbf{1},\mathbf{1}',\mathbf{1}'')$; and iii) $L_L \sim (\mathbf{1},\mathbf{1}',\mathbf{1}'')$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$. These only lead to nonvanishing charged-lepton masses when the VEV alignment is $(1,1,1)$ or $(\pm 1, \eta, \eta^*)$.

Let us consider in detail case i), where all fields are in the triplet representation. Then, Eq. (21) holds and we may parametrize

$$m_\ell = \begin{pmatrix} 0 & b\,e^{i\beta}\,v_3 & a\,e^{i\alpha}\,v_2 \\ a\,e^{i\alpha}\,v_3 & 0 & b\,e^{i\beta}\,v_1 \\ b\,e^{i\beta}\,v_2 & a\,e^{i\alpha}\,v_1 & 0 \end{pmatrix},$$
$$m_D = \begin{pmatrix} 0 & f\,e^{i\epsilon}\,v_3^* & d\,e^{i\delta}\,v_2^* \\ d\,e^{i\delta}\,v_3^* & 0 & f\,e^{i\epsilon}\,v_1^* \\ f\,e^{i\epsilon}\,v_2^* & d\,e^{i\delta}\,v_1^* & 0 \end{pmatrix}. \qquad (23)$$

Taking the VEV alignment $(1,1,1)$ or $(\pm 1, \eta, \eta^*)$, the eigenvalues of $m_\ell m_\ell^\dagger$ are

$$a^2 + b^2 + 2ab\cos(\alpha - \beta),$$
$$a^2 + b^2 - 2ab\cos(\alpha - \beta \pm \pi/3), \qquad (24)$$

which coincide with the squared masses of the charged leptons. As for the light neutrino mass matrix, we obtain for the VEV alignment $(1,1,1)$,

$$m_\nu = -M^{-1} m_D m_D^T = -M^{-1} \begin{pmatrix} x & y & y \\ y & x & y \\ y & y & x \end{pmatrix}, \qquad (25)$$

where $x = d^2\,e^{2i\delta} + f^2\,e^{2i\epsilon}$ and $y = d\,f\,e^{i(\delta+\epsilon)}$.

The eigenvalues of $m_\nu m_\nu^\dagger$ are

$$M^{-2}\left[d^2 + f^2 + 2df\cos(\delta - \epsilon)\right]^2, \qquad (26)$$
$$M^{-2}\left\{\left[d^2 + f^2 - df\cos(\delta - \epsilon)\right]^2 - 3d^2 f^2 \sin^2(\delta - \epsilon)\right\},$$

with the latter twice degenerate. This in turn implies that two light neutrinos are degenerate in mass, in contradiction with experiment. This feature remains for the VEV alignment $(\pm 1, \eta, \eta^*)$, although the expressions for the eigenvalues become more involved in that case.

We now turn to case ii) where $L_L \sim \mathbf{3}$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim (\mathbf{1},\mathbf{1}',\mathbf{1}'')$. The charged-lepton mass matrix can now be parametrized as

$$m_\ell = \begin{pmatrix} a\,e^{i\alpha}\,v_1 & b\,e^{i\beta}\,v_1 & c\,e^{i\gamma}\,v_1 \\ a\,e^{i\alpha}\,v_2 & \omega\,b\,e^{i\beta}\,v_2 & \omega^2\,c\,e^{i\gamma}\,v_2 \\ a\,e^{i\alpha}\,v_3 & \omega^2\,b\,e^{i\beta}\,v_3 & \omega\,c\,e^{i\gamma}\,v_3 \end{pmatrix}, \qquad (27)$$

where $\omega = e^{2i\pi/3}$. Taking the VEV alignments $(1,1,1)$, or $(\pm 1, \eta, \eta^*)$, the eigenvalues of $m_\ell m_\ell^\dagger$ are $3a^2$, $3b^2$, and $3c^2$, in agreement with our second requirement that there should be three massive, nondegenerate charged-lepton masses. Yet, the analysis for neutrinos is the same as the one performed in case i), thus leading to two degenerate light neutrino masses, which contradicts neutrino data.

Let us now analyze case iii), where $L_L \sim (\mathbf{1},\mathbf{1}',\mathbf{1}'')$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$. In this case, the mass matrices become

$$m_\ell = \begin{pmatrix} a\,e^{i\alpha}\,v_1 & a\,e^{i\alpha}\,v_2 & a\,e^{i\alpha}\,v_3 \\ b\,e^{i\beta}\,v_1 & \omega\,b\,e^{i\beta}\,v_2 & \omega^2\,b\,e^{i\beta}\,v_3 \\ c\,e^{i\gamma}\,v_1 & \omega^2\,c\,e^{i\gamma}\,v_2 & \omega\,c\,e^{i\gamma}\,v_3 \end{pmatrix}, \qquad (28)$$
$$m_D = \begin{pmatrix} d\,e^{i\delta}\,v_1^* & d\,e^{i\delta}\,v_2^* & d\,e^{i\delta}\,v_3^* \\ f\,e^{i\epsilon}\,v_1^* & \omega\,f\,e^{i\epsilon}\,v_2^* & \omega^2\,f\,e^{i\epsilon}\,v_3^* \\ g\,e^{i\xi}\,v_1^* & \omega^2\,g\,e^{i\xi}\,v_2^* & \omega\,g\,e^{i\xi}\,v_3^* \end{pmatrix}. \qquad (29)$$

For both VEV, $(1,1,1)$ and $(\pm 1, \eta, \eta^*)$, $m_\ell m_\ell^\dagger = 3\,\mathrm{diag}\,(a^2, b^2, c^2)$ so that we can accommodate nonvanishing nondegenerate charged leptons. Furthermore, for the VEV $(1,1,1)$, we obtain

$$m_\nu = -M^{-1} m_D m_D^T = -M^{-1} \begin{pmatrix} z & 0 & 0 \\ 0 & 0 & t \\ 0 & t & 0 \end{pmatrix}, \qquad (30)$$

where $z = 3d^2\,e^{2i\delta}$ and $t = 3fg\,e^{i(\epsilon+\xi)}$. Thus, $m_\nu m_\nu^\dagger = 9\,\mathrm{diag}\,(d^4, f^2 g^2, f^2 g^2)$, and we get two degenerate light neutrinos. The matrices for the VEV $(\pm 1, \eta, \eta^*)$ are slightly different, but the conclusions are the same. As a result, cases i), ii), and iii) are ruled out by experiment.

Now we consider the possibility that $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. From Table II in Ref. [8] we conclude that one must have $L_L \sim \mathbf{3}$, while we can have either $\ell_R \sim \mathbf{3}$ or ℓ_R in singlet representations. We find that the choice $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\ell_R \sim \mathbf{3}$, only leads to nonvanishing charged-lepton masses when the Higgs VEV is $(1,1,1)$ or $(\pm 1, \eta, \eta^*)$. Similarly, when ℓ_R are in singlet representations, only the representation choice $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\ell_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, with the VEV alignment $(1,1,1)$ or $(\pm 1, \eta, \eta^*)$, allows for nonvanishing charged-lepton masses.

For case iv) $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\ell_R \sim \mathbf{3}$, the analysis of the charged leptons coincides with that of case i). The matrix m_D can be easily inferred from Eq. (27) of case ii):

$$m_D = \begin{pmatrix} d\, e^{i\delta}\, v_1^* & f\, e^{i\epsilon}\, v_1^* & g\, e^{i\xi}\, v_1^* \\ d\, e^{i\delta}\, v_2^* & \omega\, f\, e^{i\epsilon}\, v_2^* & \omega^2\, g\, e^{i\xi}\, v_2^* \\ d\, e^{i\delta}\, v_3^* & \omega^2\, f\, e^{i\epsilon}\, v_3^* & \omega\, g\, e^{i\xi}\, v_3^* \end{pmatrix}. \tag{31}$$

Writing

$$M_R = \begin{pmatrix} r_1\, e^{i\sigma_1} & 0 & 0 \\ 0 & 0 & r_2\, e^{i\sigma_2} \\ 0 & r_2\, e^{i\sigma_2} & 0 \end{pmatrix}, \tag{32}$$

we can use Eq. (16) to determine m_ν. The expression is long, but the eigenvalues of $m_\nu\, m_\nu^\dagger$ are simply given by $(3d^2/r_1)^2$ and $(3fg/r_2)^2$, with the latter twice degenerate. Thus, in this case we also get two degenerate light neutrinos.

The analysis of the remaining case v) $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\ell_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ is straightforward. Indeed, the charged lepton sector coincides with the one of case ii), while the light neutrinos mirror those of case iv). We then conclude that this case is also excluded.

2. *Vanishing neutrino masses*

We now turn to the possibility that m_D and, thus, m_ν have determinant equal to zero, corresponding to at least one massless light neutrino. Clearly, we exclude the possibility that $m_D = 0$, because that would lead to three massless light neutrinos. We also exclude two massless neutrinos, and search for cases with only one massless neutrino. The analysis for $\nu_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$ and $L_L \sim \mathbf{3}$ follows closely the one made previously for $\nu_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. The only cases consistent with nonvanishing charged-lepton masses are: vi) $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$, $\ell_R \sim \mathbf{3}$; and vii) $L_L \sim \mathbf{3}$, $\nu_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$, $\ell_R \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, provided that the VEV alignment is $(1,1,1)$ or $(\pm 1, \eta, \eta^*)$.

In case vi), m_ℓ is the same as for case i). As for the Dirac neutrino mass matrix, we get

$$m_D = \begin{pmatrix} d\, e^{i\delta}\, v_1^* & f\, e^{i\epsilon}\, v_1^* & g\, e^{i\xi}\, v_1^* \\ d\, e^{i\delta}\, v_2^* & f\, e^{i\epsilon}\, v_2^* & g\, e^{i\xi}\, v_2^* \\ d\, e^{i\delta}\, v_3^* & f\, e^{i\epsilon}\, v_3^* & g\, e^{i\xi}\, v_3^* \end{pmatrix}. \tag{33}$$

Taking the VEV $(1,1,1)$, we find that

$$m_D = V_L\ D_D\ V_R^\dagger, \tag{34}$$

where $D_D = \sqrt{3}\, e^{i\xi} R_3\, \mathrm{diag}(0,0,1)$,

$$V_L = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \tag{35}$$

$$V_R = \begin{pmatrix} -\frac{g\, e^{-i(\delta-\xi)}}{R_2} & \frac{df\, e^{i(\epsilon-\xi)}}{R_2 R_3} & \frac{d\, e^{-i(\delta-\xi)}}{R_3} \\ 0 & -\frac{R_2\, e^{i(\delta-\xi)}}{R_3} & \frac{f\, e^{-i(\epsilon-\xi)}}{R_3} \\ \frac{d}{R_2} & \frac{fg\, e^{i(\delta+\epsilon-2\xi)}}{R_2 R_3} & \frac{g}{R_3} \end{pmatrix}, \tag{36}$$

$R_2 = (d^2+g^2)^{1/2}$ and $R_3 = (d^2+f^2+g^2)^{1/2}$. Here V_L and V_R are the unitary matrices that diagonalize the Hermitian matrices $m_D m_D^\dagger$ and $m_D^\dagger m_D$, respectively.

Using Eq. (16), we get

$$\begin{aligned} m_\nu &= -V_L\, D_D\, X\, D_D\, V_L^T \\ &= -e^{2i\xi}(d^2+f^2+g^2) X_{33} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \end{aligned} \tag{37}$$

where

$$X = V_R^\dagger M_R^{-1} V_R^*. \tag{38}$$

¿From Eq. (37), we find the eigenvalues of $m_\nu m_\nu^\dagger$ to be $m_1^2 = m_2^2 = 0$ and $m_3^2 = 9(d^2+f^2+g^2)^2 |X_{33}|^2$. Since there are two massless neutrinos, this possibility is ruled out. For the VEV alignment $(1, \eta, \eta^*)$, the intermediate steps get more involved but the eigenvalues of $m_\nu m_\nu^\dagger$ have the same expressions and, therefore, this case is also ruled out.

Finally, it is easy to see that case vii) is also inconsistent with experiment. Indeed, m_ℓ has the same matrix structure of case ii), while m_ν is the same as in case vi) that we have just analyzed.

Before concluding this section, let us comment on the type III seesaw mechanism [13]. In the type III seesaw framework, instead of three right-handed singlet neutrino fields, one adds three Majorana neutrinos, Σ_R, in the triplet representation of the gauge group $SU(2)_L$,

$$\Sigma_{iR} = \begin{pmatrix} \Sigma_i^0/\sqrt{2} & \Sigma_i^+ \\ \Sigma_i^- & -\Sigma_i^0/\sqrt{2} \end{pmatrix}, \quad i = 1,2,3. \tag{39}$$

The relevant Lagrangian is very similar to Eq. (12) for type I seesaw,

$$\begin{aligned} -\mathcal{L}_{\mathrm{II}} &= \bar{L}_L \sum_{k=1}^{3} Y_{\ell,k} \Phi_k \ell_R + \bar{L}_L \sum_{k=1}^{3} Y_{\Sigma,k}^* \tilde{\Phi}_k \Sigma_R \\ &\quad + \frac{1}{2} \sum_{i,j=1}^{3} (M_\Sigma)_{ij} \mathrm{Tr}\left(\bar{\Sigma}_{iR} \Sigma_{jR}^C\right) + \mathrm{H.c.}\ . \end{aligned} \tag{40}$$

The effective light neutrino mass matrix acquires the same seesaw structure of Eq. (16), with M_R replaced by M_Σ. As a result, the analysis of flavor structures under the A_4 symmetry is the same as before and all the conclusions hold. In particular, Table II applies with the obvious replacement $\nu_R \to \Sigma_R$.

IV. SOFTLY BROKEN A_4 SYMMETRY

We now consider the possibility that the effective operator is not invariant under A_4. This situation is well behaved as long as we guarantee that this non-invariance comes, at the UV level, from terms that do not spoil renormalizability. We therefore consider the possibility that A_4 is broken softly by dimension-three terms contributing to the right-handed neutrino mass matrix M_R [14]. In this work, we do not aim at performing an exhaustive analysis of all the different possibilities. For definiteness, we consider here the case $L_L \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, $\nu_R \sim \mathbf{3}$, $\ell_R \sim \mathbf{3}$, so that Eqs. (28) and (29) hold for the charged-lepton mass matrix m_ℓ and the Dirac neutrino mass matrix m_D, respectively. We choose the VEV alignment $(1,1,1)$. There are two important features of m_ℓ in this case. First, m_ℓ is diagonalized exclusively through a unitary transformation on the right-handed fields ℓ_R, implying that the PMNS matrix arises exclusively from the diagonalization of m_ν. Second, the eigenvalues of $m_\ell m_\ell^\dagger$ from Eq. (28) are $3a^2$, $3b^2$, and $3c^2$. Thus, we take $a = m_e/\sqrt{3}$, $b = m_\mu/\sqrt{3}$, and $c = m_\tau/\sqrt{3}$, and consider $\alpha = \beta = \gamma = 0$.

We will now assume that A_4 is softly broken in the right-handed neutrino sector, such that the form of M_R in Eq. (21) is altered. As a numerical example, we take the symmetric matrix M_R to be

$$M_R = 10^{14}\,\mathrm{GeV} \times \begin{pmatrix} 0.6622 + 7.7769\,i & -0.4304 + 0.5404\,i & -0.0490 - 0.4532\,i \\ -0.4304 + 0.5404\,i & 5.0726 + 5.9385\,i & 0.1819 + 0.1479\,i \\ -0.0490 - 0.4532\,i & 0.1819 + 0.1479\,i & 6.5927 + 4.2281\,i \end{pmatrix}, \tag{41}$$

and, for use in the matrix m_D of Eq. (29),

$$\begin{aligned} d\,e^{i\delta} &= (0.7663 + 0.2958\,i)(246\,\mathrm{GeV}), \\ f\,e^{i\epsilon} &= (0.7516 - 0.0537\,i)(246\,\mathrm{GeV}), \\ g\,e^{i\xi} &= (-0.7477 - 0.5013\,i)(246\,\mathrm{GeV}). \end{aligned} \tag{42}$$

Upon diagonalization of the neutrino mass matrix m_ν given by Eq. (16), we find a nearly degenerate normal neutrino mass spectrum with $m_1 \simeq 0.1545$ eV, $\Delta m^2_{21} = 7.46 \times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2_{31} = 2.55 \times 10^{-3}\,\mathrm{eV}^2$, which are well within the 1σ ranges given in Ref. [15].

Next, we equate the diagonalizing matrix U [see Eq. (18)] with the PMNS neutrino mixing matrix written in the form V diag$(1, e^{i\,\alpha_M/2}, e^{i\,\beta_M/2})$, where V has the standard parametrization form [16] for the Cabibbo-Kobayashi-Maskawa matrix, and α_M and β_M are Majorana phases. This leads to $\sin^2\theta_{12} = 0.336$, $\sin^2\theta_{23} = 0.613$, and $\sin^2\theta_{13} = 0.0247$. According to Ref. [15], the first value is close to its 1σ upper bound, while the other two coincide with the best-fit values. Finally, the as-yet unmeasured CP-violating phases turn out to be $\delta = 0.53\pi$, $\alpha_M = 1.79\pi$, and $\beta_M = 1.90\pi$.

In the above numerical example, the magnitudes of the entries in M_R are

$$7.83 \times 10^{14}\,\mathrm{GeV} \times \begin{pmatrix} 0.997 & 0.088 & 0.058 \\ 0.088 & 0.997 & 0.030 \\ 0.058 & 0.030 & 1.000 \end{pmatrix}. \tag{43}$$

This shows that the current neutrino oscillation data can be easily fitted in flavor models of the type discussed in this article, if the A_4 symmetry is softly broken in the heavy Majorana neutrino mass matrix M_R by coefficients that deviate perturbatively (of the order of 10% or less) from the leading A_4-symmetric terms. Notice also that in the above example a normal (quasi-degenerate) neutrino mass spectrum is obtained. Numerical examples that lead to an inverted neutrino mass hierarchy, and simultaneously to low-energy neutrino parameters in agreement with the present 1σ ranges, can be equally constructed.

V. CONCLUSIONS

In this paper we have studied the possibility of generating the neutrino masses and mixing in the context of flavor models with three scalar doublets in the triplet representation of the A_4 group and three lepton families. We have shown that none of the possible VEV alignments that corresponds to a global minimum of the scalar potential yields phenomenologically viable charged-lepton and neutrino mass matrices. In particular, there is no representation assignment that leads to a dimension-five neutrino mass operator consistent with the present oscillation data. This in turn implies that, in this minimal A_4 construction, the canonical type I (type III) seesaw mechanism is not consistent with experiment. Notice that, from the point of view of the low-energy effective operator, this conclusion holds for any number of right-handed singlet (triplet) neutrinos, since the dimension-five operator is the same regardless of the number of heavy fields.

Furthermore, since A_4 is a subgroup of S_4, our conclusions also remain valid in flavor models based on the latter group.

Finally, aiming at constructing a viable scenario, we considered as starting point the type-I seesaw mechanism with three heavy right-handed neutrinos. Through a simple numerical example we then showed that it is indeed possible to get agreement with neutrino oscillation data, if the A_4 symmetry is softly broken at the Lagrangian level by dimension-three right-handed neutrino mass terms.

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Appendix: Couplings of $L_L L_L$ when L_L is in a singlet representation of A_4

Let us consider the group structure of the combination $L_L L_L$, when L_L is an A_4 singlet. We concentrate on the group constraints, ignoring any spinor or $SU(2)_L$ characteristics. We also note that, in all cases of interest to us, the $L_L L_L$ coupling matrix *must be symmetric*. Disregarding irrelevant permutations, we must consider the cases

$$\begin{aligned} &(\mathbf{1},\mathbf{1},\mathbf{1}), \quad (\mathbf{1},\mathbf{1},\mathbf{1}'),\\ &(\mathbf{1},\mathbf{1}',\mathbf{1}'), \quad (\mathbf{1},\mathbf{1},\mathbf{1}''),\\ &(\mathbf{1},\mathbf{1}'',\mathbf{1}''), \quad (\mathbf{1},\mathbf{1}',\mathbf{1}''),\\ &(\mathbf{1}',\mathbf{1}',\mathbf{1}'), \quad (\mathbf{1}',\mathbf{1}',\mathbf{1}''),\\ &(\mathbf{1}',\mathbf{1}'',\mathbf{1}''), \quad (\mathbf{1}'',\mathbf{1}'',\mathbf{1}''). \end{aligned} \tag{A.1}$$

We can combine the couplings of $L_L L_L$ with some other group structure in a 3×3 matrix. To explain our notation, we will use the example of $L_L \sim (\mathbf{1},\mathbf{1},\mathbf{1})$. We construct the matrix of all field products

$$\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}, \tag{A.2}$$

where a matrix element $\mathbf{1}$ means that there is at that matrix position an arbitrary complex entry, if we are coupling $L_L L_L$ to some other group structure transforming like $\mathbf{1}$ of A_4, and zero, otherwise. For example, the scalar combination $(\Phi\Phi) \sim \mathbf{1}$ has couplings to all bilinears of $L_L L_L$ when $L_L \sim (\mathbf{1},\mathbf{1},\mathbf{1})$, while a $(\Phi\Phi) \sim \mathbf{1}'$ will couple to none.

As a further example, consider $L_L \sim (\mathbf{1},\mathbf{1},\mathbf{1}')$. The corresponding matrix is

$$\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1}' \\ \mathbf{1} & \mathbf{1} & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}' & \mathbf{1}'' \end{pmatrix}. \tag{A.3}$$

This means that a $(\Phi\Phi) \sim \mathbf{1}$ will only introduce couplings in the upper left 2×2 sub-matrix, a $(\Phi\Phi) \sim \mathbf{1}''$ will only have couplings to $(L_i L_3 + L_3 L_i)$ with $i = 1, 2$, while $(\Phi\Phi) \sim \mathbf{1}'$ would only couple to $L_3 L_3$. For simplicity, we wrote $L_L = (L_1, L_2, L_3)$. We also recall that $\mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}$.

The remaining possibilities in Eq. (A.1) lead to

$$\begin{pmatrix} \mathbf{1} & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}' & \mathbf{1}'' & \mathbf{1}'' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1}'' \\ \mathbf{1} & \mathbf{1} & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}' \end{pmatrix}, \tag{A.4}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}'' & \mathbf{1}' & \mathbf{1}' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1} & \mathbf{1}' & \mathbf{1}'' \\ \mathbf{1}' & \mathbf{1}'' & \mathbf{1} \\ \mathbf{1}'' & \mathbf{1} & \mathbf{1}' \end{pmatrix}, \tag{A.5}$$

$$\begin{pmatrix} \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}'' \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1}'' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1}'' & \mathbf{1}'' & \mathbf{1} \\ \mathbf{1}'' & \mathbf{1}'' & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1}' \end{pmatrix}, \tag{A.6}$$

$$\begin{pmatrix} \mathbf{1}'' & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1} & \mathbf{1}' & \mathbf{1}' \end{pmatrix}, \quad \begin{pmatrix} \mathbf{1}' & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}' & \mathbf{1}' \\ \mathbf{1}' & \mathbf{1}' & \mathbf{1}' \end{pmatrix}, \tag{A.7}$$

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